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ROTATIONAL BANDS FOR EVEN-EVEN NUCLEI IN THE FRAME OF SOOD MODEL

H. O. NAFIE, JAMAL H. MADANI AND K. A. GADO
Department of physics, Faculty of science, Tabuk University, KSA

Abstract

Using the theory of molecular spectra to obtain an estimate of the higher- order corrections to the energy levels of a non-rigid rotator, we sum the infinite power series in $I(I+1)$ to describe the energy levels in the ground-state rotational bands of deformed even-even nuclei. The predictions of the Sood model formula show surprisingly good agreement with the experimentally observed energy levels in even-even nuclei in the $100 \leq A \leq 160$ region.

Keywords: Rotational bands, $100 \leq A \leq 160$

1. INTRODUCTION

Deformed nuclei, those characterized by a non-spherical spatial distribution of nuclear density are known to exhibit rotational bands in their spectra [1], microscopic description of the rotational motion involves coherent contributions from many nucleons and it's thus referred to as a collective motion. That results in a rotation of the nucleus as a whole around an axis different from the nuclear symmetry axis. A schematic example of a collective rotation is a prolate-nucleus that rotates around an axis perpendicular to the nuclear symmetry axis. It was found, from experimental spectra, that the relation between the excitation energy E and spin I is often a smooth one while for spins that are not too high, it can be approximated by $E \sim I(I+1)$. The corresponding series of states with consecutively increasing angular momentum is called a rotational band. The lowest state of a band is referred to as bandhead. Many states of different intrinsic structure can in principle become bandheads, the band build on the ground state of the nucleus is referred to as the ground state band (GRB). All others bands are called excited bands or side bands. The lowest energy state of a given angular momentum is called the yrast state [2]. The ground state excitation energy calculation is given in section 2. A brief discussion of our obtained results in comparison to the Cluster- Core model calculations and experimental data is given in section 3.

2. THE SOOD MODEL

According to the Bohr-Mottelson (1975) the lowest rotational energy levels for nuclei are given by the formula [3]:

$$E(I) = A[I(I+1)] = \frac{\hbar^2}{2\vartheta}[I(I+1)] \dots (2-1)$$

$A = \frac{1}{2\vartheta}$, where ϑ is the nuclear moment of inertia, and I is the angular momentum follows sequence $0^+, 2^+, 4^+, \dots$.

Equation $(2-1)$, considered the nucleus as rigid rotator, while actually the shape of the nucleus depends on the angular momentum, due to the centrifugal stretching or rotation vibration interaction.

So, the rotational energy can be written as infinite series Nielson [4], Lipas [5], Xu. et. al [6], etc….

$$\mathrm{E(I)} = \mathrm{A}[\mathrm{I}(\mathrm{I}+1)] - \mathrm{B}[\mathrm{I}(\mathrm{I}+1)]^2 + \mathrm{C}[\mathrm{I}(\mathrm{I}+1)]^3 - \mathrm{D}[\mathrm{I}(\mathrm{I}+1)]^4 + \cdots \qquad (2-2)$$

Where $A, B, C, and\ D$ are fitting parameters. Rewrite equation $(2\ -\ 2)$, as follow:

$$\mathrm{E(I)} = \mathrm{A}[\mathrm{I}(\mathrm{I}+1)]\left\{1 - \left(\frac{B}{A}\right)I(I+1)\left[1 - \left(\frac{C}{B}\right)I(I+1) + \left(\frac{D}{B}\right)\left(I(I+1)\right)^2 \ldots\right]\right\}\ldots \qquad (2-3)$$

Following Sood [7], one may write $\frac{C}{B} = N\left(\frac{B}{A}\right)$, where N is of order 2 to 3 (From molecular spectra theory Dunham [8] (1967) and one know that $\frac{D}{B}$ is order $\left(\frac{C}{B}\right)^2$, Xu.et al [6] (1989)).

Rewrite equation $(2\ -\ 3)$ in the form:

$$\mathrm{E(I)} = \mathrm{A}[\mathrm{I}(\mathrm{I}+1)]\left\{1 - \left(\frac{B}{A}\right)I(I+1)\left[1 - N\left(\frac{B}{A}\right)I(I+1) + \left(N\left(\frac{B}{A}\right)I(I+1)\right)^2 - \cdots\right]\right\}\ldots \qquad (2-4)$$

The terms between in square bracket in the above equation is a geometric series, then using the sum rule of the geometric series one gets:

$$\mathrm{E(I)} = \mathrm{A}[\mathrm{I}(\mathrm{I}+1)]\left\{1 - \frac{\left(\frac{B}{A}\right)I(I+1)}{1 + N\left(\frac{B}{A}\right)I(I+1)}\right\} = A(I)[I(I+1)]\ldots \quad (2-5)$$

Which gives the analytical expression for the moment of inertia in terms of angular momentum I. N Parameter is taken from Sood in the form [9]:

$$N = 2.85 - 0.05I \ldots \qquad (2-6)$$

3. RESULTS AND DISCUSSION:

In an attempt to obtain better agreement with experimental, we used equation $(2-5)$, where we disregard in the first place the relative motions of the nuclei. A diatomic molecule, so far as its mass distribution is concerned, can be pictured as a nearly rigid dumb-bell [10], since of course the electrons by reason of their vanishingly small mass form an inappreciable factor in the mass distribution. The energy levels predictions are shown in the seventh column in table (1) as compared with the recent experimental know data for nuclei. A good agreement is shown.

We have calculated the rotational spectrum for the ground state band up to $I^{\pi} = 12^{+}$ for nuclei ^{102}Zr, ^{110}Pd, ^{112}Cd and ^{120}Ba, while up to $I^{\pi} = 14^{+}$ for ^{100}Zr, ^{122}Xe, $^{130\text{-}132}$Nd, ^{142}Gd, $^{150\text{-}152}$Nd and ^{160}Dy[11-16].

The calculated energies for the chosen nuclei are presented in table (1). In this table, the first and second columns contain the angular momenta and experimental energies. The last row represents the chi-square test of the predicted energies from the experimental ones. The third, the fourth, the fifth and the sixth columns contain the present predicted energies for the four form potentials cluster-core model[17], the last column contains the energy predicted values according to equation (2-5). In the Sood model equation (2-5); extracting the ratio $R_4 = \frac{E_4}{E_2}$ from the experimental data, one can determine a reasonable value for the parameter $\frac{B}{A}$. Then, the value of the parameter A is determined from the experimental value of E_4 or E_2. The comparison of our calculations and the experimental data [11-16] for typical nuclei is given in table (1). Comparing the predicted energies given by Sood model and cluster model, for nuclei ^{102}Zr, ^{120}Ba, ^{122}Xe, $^{130,\ 150\text{-}152}$ Nd and ^{160}Dy,with the corresponding experimental data, one notices that the results of Sood model are better than the results of cluster model. While for the two nuclei ^{100}Zr and ^{132}Nd the predicted energies are approximately the same for the two models. Therefore, the studies indicate that, the Sood model may provide an accurate description for the considered nuclei.

CONCLUSION

In this work, the ground state band of the considered heavy deformed nuclei are studied. The energy levels of the even-even nuclei are calculated. From this study we conclude that, the Sood model predict the excitation energies of the ground-state band of the considered nuclei, which have mass number in the range; $(100 \leq A \leq 160)$. Finally, in general the predicated values for Sood model are in good agreement with the experimental data rather than other cluster-core model. In spite the number of parameters in Sood model are less than the number of parameters in cluster model, the Sood model described the energies better than the cluster model for the chosen nuclei

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Table 1: The theoretical energy levels comparison with the experimental data and Cluster model for the considered even-even nuclei

^{100}Zr Nucleus						
Spin I^{π}	Experimental	Energy Levels (Mev)				
		By Cluster- Core Model				By Sood Model
		Modified Woods-Saxon	Symmetric Woods-Saxon	Pure Woods-Saxon	Gaussian	
2^+	0.213	0.100	0.122	0.208	0.330	0.213
4^+	0.564	0.757	0.560	0.720	0.599	0.564
6^+	1.051	1.389	1.067	1.217	0.873	1.021
8^+	1.687	2.010	1.701	1.779	1.261	1.574
10^+	2.426	2.589	2.514	2.418	1.849	2.216
12^+	3.272	3.054	3.500	3.155	2.790	2.933
14^+	4.209	3.307	4.686	3.974	4.446	3.708
Chi-square test		0.73	0.16	0.86	0.60	0.53

^{102}Zr Nucleus						
Spin I^{π}	Experimental	Energy Levels (Mev)				
		By Cluster- Core Model				By Sood Model
		Modified Woods-Saxon	Symmetric Woods-Saxon	Pure Woods-Saxon	Gaussian	
2^+	0.152	0.089	0.082	0.152	0.200	0.152
4^+	0.478	0.746	0.522	0.639	0.470	0.478
6^+	0.965	1.374	1.031	1.090	0.743	0.978
8^+	1.595	1.977	1.667	1.595	1.132	1.612
10^+	2.352	2.543	2.482	2.148	1.722	2.393
12^+	3.212	3.005	3.474	2.779	2.665	3.289
Chi-square test		0.78	0.12	1.14	0.77	0.01

^{110}Pd Nucleus						
Spin I^{π}	Experimental	Energy Levels (Mev)				
		By Cluster- Core Model				By Sood Model
		Modified Woods-Saxon	Symmetric Woods-Saxon	Pure Woods-Saxon	Gaussian	
2^+	0.374	0.335	0.576	0.330	0.489	0.374
4^+	0.921	1.001	0.986	0.930	0.868	0.921
6^+	1.574	1.629	1.436	1.576	1.307	1.631
8^+	2.296	2.226	2.008	2.333	1.936	2.494
10^+	3.131	2.783	2.716	3.246	2.859	3.494
12^+	4.030	3.233	3.587	4.308	4.332	4.607

^{110}Pd Nucleus						
Spin I^{π}	Experimental	Energy Levels (Mev)				
		By Cluster- Core Model				By Sood Model
		Modified Woods-Saxon	Symmetric Woods-Saxon	Pure Woods-Saxon	Gaussian	
Chi-square test		0.35	0.44	0.41	0.31	0.71

^{112}Cd Nucleus						
Spin I^{π}	Experimental	Energy Levels (Mev)				
		By Cluster- Core Model				By Sood Model
		Modified Woods-Saxon	Symmetric Woods-Saxon	Pure Woods-Saxon	Gaussian	
2^+	0.618	0.632	1.033	0.755	0.932	0.618
4^+	1.416	1.302	1.438	1.349	1.321	1.416
6^+	2.168	1.931	1.865	1.969	1.782	2.383
8^+	2.881	2.526	2.404	2.697	2.442	3.511
10^+	3.684	3.092	3.080	3.548	3.414	4.779
12^+	4.587	3.531	3.915	4.567	4.948	6.167
Chi-square test		0.69	0.99	0.77	0.58	4.41

^{120}Ba Nucleus						
Spin I^{π}	Experimental	Energy Levels (Mev)				
		By Cluster- Core Model				By Sood Model
		Modified Woods-Saxon	Symmetric Woods-Saxon	Pure Woods-Saxon	Gaussian	
2^{+}	0.186	0.121	0.135	0.194	0.170	0.186
4^{+}	0.544	0.840	0.490	0.718	0.542	0.544
6^{+}	1.040	1.447	0.850	1.191	0.979	1.025
8^{+}	1.645	2.064	1.279	1.712	1.597	1.622
10^{+}	2.336	2.617	1.818	2.299	2.521	2.319
12^{+}	3.083	3.068	2.493	3.001	3.997	3.010
Chi-square test		0.80	0.60	0.98	0.48	0.01

^{122}Xe Nucleus						
Spin I^{π}	Experimental	Energy Levels (Mev)				
		By Cluster- Core Model				By Sood Model
		Modified Woods-Saxon	Symmetric Woods-Saxon	Pure Woods-Saxon	Gaussian	
2^{+}	0.331	0.353	0.563	0.275	0.462	0.331
4^{+}	0.828	1.037	0.940	0.834	0.796	0.828
6^{+}	1.467	1.659	1.329	1.369	1.197	1.460
8^{+}	2.217	2.272	1.808	1.936	1.765	2.215
10^{+}	3.040	2.827	2.414	2.636	2.615	3.078
12^{+}	3.919	3.274	3.159	3.422	3.984	4.031
14^{+}	4.900	3.533	4.081	4.377	5.802	5.050

^{122}Xe Nucleus						
Spin I^{π}	**Experimental**	**Energy Levels (Mev)**				
		By Cluster- Core Model				**By Sood Model**
		Modified Woods-Saxon	**Symmetric Woods-Saxon**	**Pure Woods-Saxon**	**Gaussian**	
Chi-square test		0.83	0.97	0.76	0.60	0.02

^{130}Nd Nucleus						
Spin I^{π}	**Experimental**	**Energy Levels (Mev)**				
		By Cluster- Core Model				**By Sood Model**
		Modified Woods-Saxon	**Symmetric Woods-Saxon**	**Pure Woods-Saxon**	**Gaussian**	
2^+	0.159	0.009	0.206	0.160	0.277	0.159
4^+	0.486	0.713	0.567	0.680	0.509	0.486
6^+	0.940	1.343	0.893	1.112	0.723	0.917
8^+	1.487	1.956	1.303	1.600	1.016	1.449
10^+	2.100	2.515	1.811	2.135	1.488	2.070
12^+	2.764	2.970	2.449	2.763	2.270	2.764
14^+	3.468	3.248	3.261	3.533	3.701	3.513
Chi-square test		0.97	0.20	0.99	0.81	0.02

^{132}Nd Nucleus						
Spin I^{π}	Experimental	Energy Levels (Mev)				
		By Cluster- Core Model				By Sood Model
		Modified Woods-Saxon	Symmetric Woods-Saxon	Pure Woods-Saxon	Gaussian	
2^+	0.213	0.182	0.267	0.169	0.343	0.213
4^+	0.610	0.884	0.627	0.688	0.598	0.610
6^+	1.131	1.522	0.960	1.125	0.845	1.121
8^+	1.710	2.140	1.374	1.620	1.190	1.742
10^+	2.309	2.701	1.883	2.156	1.723	2.461
12^+	2.945	3.159	2.531	2.792	2.608	3.260
14^+	3.630	3.445	3.351	3.566	4.134	4.120
Chi-square test		0.66	0.37	0.75	0.81	0.49

^{140}Ba Nucleus						
Spin I^{π}	Experimental	Energy Levels (Mev)				
		By Cluster- Core Model				By Sood Model
		Modified Woods-Saxon	Symmetric Woods-Saxon	Pure Woods-Saxon	Gaussian	
2^+	0.602	0.599	0.688	0.552	0.760	0.602
4^+	1.131	1.312	1.049	1.155	1.064	1.131
6^+	1.661	1.919	1.433	1.809	1.400	1.615
8^+	2.469	2.535	1.876	2.566	1.867	2.055
10^+	3.384	3.089	2.450	3.477	2.576	2.456
Chi-square test		0.19	0.89	0.52	0.85	0.27

^{142}Gd Nucleus						
Spin I^{π}	Experimental	Energy Levels (Mev)				
		By Cluster- Core Model				By Sood Model
		Modified Woods-Saxon	Symmetric Woods-Saxon	Pure Woods-Saxon	Gaussian	
2^{+}	0.515	0.569	0.877	0.538	0.767	0.515
4^{+}	1.209	1.277	1.263	1.150	1.099	1.209
6^{+}	2.003	1.939	1.660	1.750	1.483	2.014
8^{+}	2.759	2.554	2.151	2.421	2.038	2.926
10^{+}	3.409	3.124	2.774	3.204	2.856	3.933
12^{+}	4.103	3.585	3.552	4.144	4.197	5.018
14^{+}	4.902	3.884	4.524	5.275	5.889	6.162
Chi-square test		0.46	0.95	0.69	0.99	2.04

^{150}Nd Nucleus						
Spin I^{π}	Experimental	Energy Levels (Mev)				
		By Cluster- Core Model				By Sood Model
		Modified Woods-Saxon	Symmetric Woods-Saxon	Pure Woods-Saxon	Gaussian	
2^{+}	0.135	0.001	0.060	0.134	0.125	0.135
4^{+}	0.381	0.497	0.420	0.559	0.383	0.381
6^{+}	0.720	1.004	0.745	0.884	0.633	0.714
8^{+}	1.130	1.475	1.153	1.199	0.983	1.127
10^{+}	1.599	1.853	1.656	1.530	1.522	1.609

^{150}Nd Nucleus						
Spin I^{π}	Experimental	Energy Levels (Mev)				
		By Cluster- Core Model				By Sood Model
		Modified Woods-Saxon	Symmetric Woods-Saxon	Pure Woods-Saxon	Gaussian	
12^+	2.119	2.145	2.294	1.909	2.418	2.149
14^+	2.683	2.272	3.103	2.376	3.950	2.733
Chi-square test		0.70	0.18	1.45	0.96	0.01

^{152}Nd Nucleus						
Spin I^{π}	Experimental	Energy Levels (Mev)				
		By Cluster- Core Model				By Sood Model
		Modified Woods-Saxon	Symmetric Woods-Saxon	Pure Woods-Saxon	Gaussian	
2^+	0.073	0.001	0.001	0.001	0.061	0.073
4^+	0.237	0.393	0.231	0.259	0.276	0.237
6^+	0.484	0.771	0.564	0.548	0.442	0.484
8^+	0.805	1.105	0.970	0.817	0.689	0.804
10^+	1.195	1.319	1.487	1.105	1.083	1.190
12^+	1.648	1.440	2.124	1.410	1.772	1.630
14^+	2.158	1.426	2.938	1.806	3.090	2.110
Chi-square test		0.99	0.87	2.31	0.64	0.01

^{160}Dy Nucleus						
Spin I^{π}	**Experimental**	**Energy Levels (Mev)**				
		By Cluster- Core Model				**By Sood Model**
		Modified Woods-Saxon	**Symmetric Woods-Saxon**	**Pure Woods-Saxon**	**Gaussian**	
2^+	0.087	0.001	0.041	0.001	0.079	0.087
4^+	0.284	0.442	0.370	0.332	0.315	0.284
6^+	0.581	0.852	0.644	0.683	0.534	0.580
8^+	0.967	1.212	0.972	1.035	0.841	0.966
10^+	1.429	1.468	1.381	1.420	1.321	1.430
12^+	1.952	1.632	1.907	1.862	2.130	1.959
14^+	2.515	1.682	2.606	2.422	3.557	2.536
Chi-square test		0.98	0.09	2.14	0.68	0.01

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